Exam. Code : 211001
Subject Code : 5473

M.Sc. Mathematics 1st Semester

COMPLEX ANALYSIS

Paper—MATH-552

Time Allowed—3 Hours] [Maximum Marks—100

Note :—Attempt **TWO** questions from each unit. All questions carry equal marks.

UNIT—I

1. Show that continuity is a necessary but not a sufficient condition for the existence of a finite derivative.

2. Examine the nature of the function

$$f(z) = \frac{x^2y^5(x+iy)}{x^4+y^{10}}, z \neq 0, f(0) = 0$$

in the region including the origin.

3. An electrostatic field in xy – Plane is given by the potential function $\phi = 3x^2 y - y^3$, find the stream function.

4. If $w = f(z)$ is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \log |f'(z)| = 0$$

If $|f'(z)|$ is the product of a function of x and function of y, show that $f'(z) = \exp(\alpha z^2 + \beta z + \gamma)$ where α is real and β and γ are complex constants.

UNIT—II

5. Define complex line integral and evaluate $\int_{-i}^{i} |z|\,dz$ along the right half of the unit circle $|z| = 1$ described in the counter — clockwise direction.

6. State and prove Cauchy's integral theorem.

7. If $f|z|$ is analytic in a region including the circle $|z| \leq R$, prove that for $0 < r < R$

$$f(re^{i\theta}) = \frac{1}{2\pi}\int_{0}^{2\pi} \frac{(R^2 - r^2) f(Re^{i\varphi})\,d\varphi}{R^2 - 2Rr\cos(\theta - \varphi) + r^2}.$$

where $a = re^{i\theta}$ is any point of the domain $|z| < R$.

8. State and prove Liouville's theorem.

UNIT—III

9. State Laurent's theorem and prove its Uniqueness.

10. State and prove minimum modulus principle.

11. If all the zeros of a polynomial lie in a half plane. Then all the zeros of derivative also lie in the same half plane.

12. State Argument principle. Use Rouche's theorem to find the number of zeros of the polynomial $2z^4 - 2z^3 + z^2 + 11$ inside the circle $|z| = 1$.

UNIT—IV

13. Define residue of a function f | z | at z = a. Find the residue of $z^3 / (z-1)^4 (z-2) (z-3)$ at the poles of the function.

14. State and prove Cauchy's residue theorem.

15. Evaluate $\int_0^{\pi} \frac{\cos 2\theta}{1-2a\cos\theta+a^2} d\theta$.

16. Prove that if $0 < a < 1$, then

$$\int_0^{\infty} \frac{x^{a-1}}{1+x} dn = \frac{\pi}{\sin a\pi}.$$